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## Scattering amplitudes at multi TeV energies\*

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### Abstract

We show that a generalized Regge behaviour,

$$\text{Im } F(s, t) \simeq \Phi(t) (\log s/\hat{s})^{\nu(t)} (s/\hat{s})^{\alpha_P(t)}, \quad \text{for } |t| < |t_0|, \quad s \rightarrow \infty$$

where  $\Phi(t) \simeq e^{bt}$ ,  $\alpha_P(t) \simeq \alpha_P(0) + \alpha'_P(0)t$ , and  $t_0$  is the first zero of  $\alpha_P(t)$ ,  $\alpha_P(t_0) = 0$ , implies that the corresponding cross section is bounded by

$$\sigma_{\text{tot}}(s) < (\text{Const.}) \times \log s/\hat{s}.$$

This growth, however, is not sufficient to fit the experimental cross sections. If, instead of this, we assume saturation of the improved Froissart bound, i.e., a behaviour

$$\text{Im } F(s, 0) \simeq A(s/\hat{s}) \log^2 \frac{s}{s_1 \log^{7/2} s/s_2},$$

a good fit is obtained to  $\pi\pi$ ,  $\pi N$ ,  $KN$  and  $NN$  cross sections from c.m. kinetic energy  $E_{\text{kin}} \simeq 1$  GeV to 30 TeV (producing a cross section of  $108 \pm 6$  mb at LHC energy). This suggests that the Regge-type behaviour only holds for values of the momentum transfer very near zero.

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\* Dedicated to Prof. Yuri Simonov, one of the first physicists to establish the connection between Regge theory and QCD.

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## 1. Introduction

It has been known for a long time<sup>[1,2]</sup> that the analyticity and unitarity properties for scattering amplitudes that follow from local field theory (or, more generally, in a theory where the *observables* are local<sup>[2]</sup>) imply the Froissart bound for total hadronic cross sections,  $A + B \rightarrow \text{all}$ ,

$$\sigma_{AB}(s) \underset{s \rightarrow \infty}{\lesssim} C \log^2 s/s_0. \quad (1.1a)$$

In particular, for  $\pi\pi$  scattering, we can calculate the constants  $C$ ,  $s_0$  and the corrections to (1.1a) for finite energies, the last in terms of the pion mass,  $m_\pi$ , and the D wave scattering lengths.<sup>[3]</sup> In fact, (1.1a) can be somewhat improved in the sense that one can show<sup>[4]</sup> that  $s_0$  must grow as a  $7/2$  power<sup>1</sup> of a logarithm of the energy, so one has the bound

$$\sigma_{AB}(s) \underset{s \rightarrow \infty}{\lesssim} C \log^2 \frac{s}{s_1 \log^{7/2} s/s_2}, \quad (1.1b)$$

and  $s_1, s_2$  are now constants.

Experimental cross sections seem to be close to the bounds (1.1) in the sense that they exhibit growth<sup>[5]</sup> at very high ( $s^{1/2} > 10$  GeV) energies. But it has up till now not been possible to calculate the *behaviour* (as opposed to mere *bounds*) for the cross sections from first principles.

It is also known that, on the other hand, a behaviour of Regge type for the (imaginary part of the) scattering amplitude,

$$\text{Im } F(s, t) \underset{s \rightarrow \infty}{\simeq} \Phi(t) (\log s/\hat{s})^{\nu(t)} (s/\hat{s})^{\alpha_P(t)}, \quad (1.2)$$

where, for very small values of  $t$ ,

$$\Phi(t) \simeq e^{bt}, \quad \alpha_P(t) \simeq \alpha_P(0) + \alpha'_P(0)t,$$

cannot hold for large values of the momentum transfer  $|t|$ . Here we expect the Brodsky–Farrar behaviour:<sup>[6]</sup>

$$\text{Im } F(s, t) \sim f(\cos \theta) s^{-p}, \quad |t| \sim s, \quad (1.3)$$

with  $p$  is related to the number of constituents in particles  $A, B$ ; for pion-pion scattering,  $p = 6$ . What is, however, not known is how the transition between the regimes described by (1.2) and (1.3) takes place.

In the present note we investigate the consequences of two possible assumptions that will allow us to make predictions for the high energy cross sections. First, we make what appears a reasonable assumption, that we will call *extended Regge* behaviour, suggested by Regge theory; this is given by Eq. (1.2), assuming smooth behaviour of the trajectory  $\alpha_P(t)$  and that (1.2) is valid up to  $|t| \sim$  a few  $\text{GeV}^2$ ; see below for precise details. This would allow us to refine the Froissart bound to the bounds

$$\frac{\text{Const.}}{\log s/s_0} \underset{s \rightarrow \infty}{\lesssim} \sigma_{\pi\pi}(s) \underset{s \rightarrow \infty}{\lesssim} (\text{Const.}) \log s/s_0 \quad [\text{Extended Regge}] \quad (1.4)$$

for the pion-pion scattering amplitudes. If, moreover, we further assume that partial wave amplitudes at fixed angular momentum,  $l$ , are mostly inelastic at high energy (actually, it is enough to assume that *one* wave is inelastic) then we could improve the bound for the cross section, getting

$$\lim_{s \rightarrow \infty} \sigma_{\pi\pi}(s)/\log s = 0 \quad [\text{Extended Regge}]. \quad (1.5)$$

This bounds may be translated, via factorization,<sup>[7]</sup> and adding the subleading  $\rho$  and  $P'$  Regge poles (necessary at –relatively– low energies) to the behaviour of  $\pi N$ ,  $KN$  and  $NN$  cross sections at high energies. If we do this, it turns out that the growth allowed by (1.4) is not very compatible with experimental cross sections at accessible energies, even if saturated, as it gives a largish  $\chi^2/\text{d.o.f.} \simeq 1.3$ . The predicted cross section at the LHC would be

$$\sigma_{pp}(s) = ((14 \text{ TeV})^2) \sim 95.5 \pm 4 \text{ mb} \quad [\text{Extended Regge}] \quad (1.6)$$

<sup>1</sup> Note that in ref. 4 the power is wrongly given as 7 instead of the correct value,  $7/2$ .

where the error is only statistical (from fit to data). This result is difficult to believe; although compatible within errors, the number in (1.6) is clearly below experiment<sup>[5]</sup> already at 6 TeV. We will discuss in Sect. 6 the reasons for the failure of (1.5), essentially due to failure of the Regge behaviour when  $t$  is not near zero, as one must have functions  $\alpha_s(t)$ ,  $\Phi(t)$  very different from what one expects in standard Regge theory.

Then we consider a second possibility, which is that the bound (1.1b) is *saturated*; we will give reasons that makes this saturation plausible. In this case, the fits to  $NN$ ,  $\pi N$  data improve clearly, and the prediction for the LHC cross section is

$$\sigma_{pp}(s) = ((14 \text{ TeV})^2) = 108 \pm 4 \pm 4 \text{ mb} \quad [\text{Saturated bound}]. \quad (1.7)$$

Here the first error is statistical (from fit to data) and the second is the estimated theoretical error. The LHC data should be able to differentiate unambiguously between this and the result from the extended Regge hypothesis, Eq. (1.6).

## 2. High energy cross sections with extended Regge behaviour

We will consider the  $\pi^0\pi^+$  scattering amplitude, to avoid inessential complications associated with spin, isospin or identity of particles. Because of unitarity, we may write the scattering amplitude as

$$F_{\pi^0\pi^+}(s, t) = \sum_l (2l+1) P_l(\cos\theta) f_l(s), \quad f_l(s) = \frac{2s^{1/2}}{\pi k} \frac{\eta_l(s) e^{2i\delta_l(s)} - 1}{2i}; \quad \cos\theta = 1 + \frac{2t}{s - 4\mu^2}. \quad (2.1)$$

The elasticity parameter is such that  $0 \leq \eta_l \leq 1$ . The new ingredient we use to get the bounds is that, in QCD, we have the Jackson–Farrar<sup>[6]</sup> behaviour for the form factor of the pion,  $F_\pi$ ,

$$F_\pi(s) \underset{s \rightarrow \infty}{\simeq} \frac{12\pi f_\pi^2 \alpha_s(|s|)}{-s}, \quad C_F = \frac{4}{3}. \quad (2.2)$$

If we call  $\delta_\pi(s)$  to the phase of  $F_\pi(s)$ , this implies that

$$\delta_\pi(s) \underset{s \rightarrow \infty}{\simeq} \pi \left\{ 1 + \frac{1}{\log s/\Lambda^2} \right\}. \quad (2.3)$$

On the other hand, the Fermi–Watson final state theorem implies that, if the inelasticity is negligible,  $\delta_\pi$  and the P wave phase,  $\delta_1$  are equal:

$$\delta_\pi \simeq \delta_1, \text{ if } \eta_1 \simeq 1.$$

Now comes the *extended Regge* assumption: we assume the behaviour

$$\text{Im } F_{\pi^0\pi^+}(s, t) \underset{s \rightarrow \infty}{\simeq} \frac{1}{3} \Phi(t) (\log s/\hat{s})^{\nu(t)} (s/\hat{s})^{\alpha_P(t)} + (\text{Const.}). \quad (2.4)$$

The factor  $1/3$  is a Clebsch–Gordan coefficient, that we separate off  $\Phi$  for convenience. We take  $\hat{s} = 1 \text{ GeV}^2$ ; the results, of course, are independent of this choice. This behaviour is suggested by Regge theory. From general conditions it follows that the functions  $\Phi(t)$ ,  $\nu(t)$  and  $\alpha_P(t)$  must be analytic functions of  $t$  in the Martin–Lehmann ellipse. Moreover, from the positivity properties of  $\text{Im } F_{\pi^0\pi^+}(s, t)$  we expect  $\alpha_P(t)$ ,  $\nu(t)$ ,  $\Phi(t)$  and all their derivatives to be positive at  $t = 0$ .

We assume that  $\alpha_P(t)$  is monotonously decreasing as  $t$  becomes more and more negative, and that this happens for all values of  $t$  provided  $|t| \ll s$ : this is the “extended” hypothesis. In fact, it is sufficient to demand that this decrease occurs for values  $|t| < |\tau_0|$ , where  $\tau_0$  is such that the integral

$$\int_{-\infty}^{-\tau_0} \text{Im } F_{\pi^0\pi^+}(s, t) dt$$

is negligible, at large  $s$ . This hypothesis is, of course, verified if one had

$$\Phi(t) \simeq e^{bt}, \quad \alpha_P(t) \simeq \alpha_P(0) + \alpha'_P(0)t,$$

up to the value  $t_0$  such that  $\alpha_P(t_0) = 0$ . From standard Regge fits, one expects  $|t_0| \sim 5 \text{ GeV}^2$ .

*Bounds.* Integrating the imaginary part of (2.4) with  $\frac{1}{2} \cos \theta$ , we get the equality for the high energy P wave

$$\frac{4}{\pi} \frac{1 - \eta_1 \cos 2\delta_1}{2} \underset{s \rightarrow \infty}{\simeq} \frac{1}{2s} \int_{-\infty}^0 dt \Phi(t) (\log s/\hat{s})^{\nu(t)} (s/\hat{s})^{\alpha_P(t)} \underset{s \rightarrow \infty}{\simeq} \frac{1}{3\alpha'_P(0)} \Phi(0) (\log s/\hat{s})^{\nu(0)-1} (s/\hat{s})^{\alpha_P(0)-1}. \quad (2.5)$$

We have, in Eq. (2.5), integrated with the formula (2.4) for all values of  $t$ . The fact that we can neglect the integral for large, negative values of  $t$  is a consequence of the *extended* Regge assumption: with it, the integral from any fixed  $-\tau_0$  to  $-\infty$  becomes negligible compared to the rest.

Because the l.h.s in (2.5) is bounded, it follows that the r.h.s. must also be bounded and hence one must have  $\alpha_P(0) \leq 1$ ,  $\nu(0) \leq 1$  and, since

$$\sigma_{\pi^0\pi^+}(s) \sim s^{-1} \text{Im } F_{\pi^0\pi^+}(s, 0),$$

we get a first improvement of the Froissart bound:

$$\sigma_{\pi^0\pi^+}(s) \underset{s \rightarrow \infty}{\lesssim} (\text{Const.}) \log s/\hat{s}. \quad (2.6a)$$

But we have more: if one had  $\nu(0) < 1$ , then the r.h.s of (2.5) would tend to zero. So, the l.h.s. would also vanish which is only possible if  $\eta_1 = 1$ . In this case, the Fermi–Watson theorem applies and (2.3) gives

$$\frac{2}{\pi} \left\{ 1 - \eta_1 + 2\pi^2 \left( \frac{1}{\log s/\Lambda^2} \right)^2 \right\} \underset{s \rightarrow \infty}{\simeq} \frac{1}{3\alpha'_P(0)} \Phi(0) (\log s/\hat{s})^{\nu(0)-1} (s/\hat{s})^{\alpha_P(0)-1}.$$

Because  $1 - \eta_1$  is positive, this is only possible if  $\alpha_P(0) \geq 1$  and  $\nu(0) - 1 \geq -2$  and we get the lower bound

$$\sigma_{\pi^0\pi^+}(s) \underset{s \rightarrow \infty}{\gtrsim} \frac{\text{Const.}}{\log s/\hat{s}}, \quad (2.6b)$$

which completes the lower and upper bound announced in (1.2).

In fact, if we assumed that the partial wave amplitudes are mostly inelastic at high energy (actually, it is enough to assume that only one wave is inelastic) it follows that the upper bound (2.6a) cannot be saturated as one must have

$$\lim_{s \rightarrow \infty} \sigma_{\pi^0\pi^+}(s) / \log s = 0.$$

### 3. Saturated Froissart-like bound

We start by a brief derivation of the bound (1.1b). We write the Froissart–Gribov representation for the D wave in  $\pi^+\pi^- \rightarrow \pi^0\pi^0$  scattering,  $f_2(t)$ , for  $0 < t \leq 4m_\pi^2$ :

$$f_2(t) = \frac{1}{k_t^2} \frac{1}{\pi} \int_{4\mu^2}^{\infty} ds \text{Im } F_{\pi^0\pi^+}(s, t) Q_2 \left( \frac{s}{2k_t^2} + 1 \right), \quad k_t = \frac{\sqrt{t - 4\mu^2}}{2}. \quad (3.1)$$

Because of elastic unitarity, it follows<sup>[4,8]</sup> that the quantity  $h(t) = f_2(t)/k_t^4$  and its two first derivatives at  $t = 4m_\pi^2$  are finite. For the second derivative, this implies a sum rule of the form

$$\int_{4\mu^2}^{\infty} ds \frac{\partial^2 \text{Im } F_{\pi^0\pi^+}(s, t) / \partial t^2|_{t=4m_\pi^2}}{s^3} = C, \quad (3.2)$$

and the constant  $C$  can be expressed in terms of the scattering length and the two first effective radii of the D wave for  $\pi^+\pi^- \rightarrow \pi^0\pi^0$  scattering. Because the derivatives of the Legendre polynomials  $P_l(\cos \theta)$  are positive for  $\cos \theta \geq 1$  ( $t \geq 0$ ), and grow rapidly with  $l$ , we get a bound for  $\text{Im } F_{\pi^0\pi^+}$ . This comes about as follows: the convergence of partial wave expansion,

$$\text{Im } F_{\pi^0\pi^+}(s, t) = \sum_l (2l+1) P_l(\cos \theta) \text{Im } f_l(s),$$

together with the fact that the  $\text{Im } f_l(s)$  are positive and bounded by  $s^{1/2}/k\pi$ , will allow us to translate (3.2) into the bound (1.1a) for the cross section. That (3.2) is finite implies that, for large,  $s$ ,

$$\left. \frac{\partial^2 \text{Im } F_{\pi^0\pi^+}(s, t)}{\partial t^2} \right|_{t=4m_\pi^2} = \frac{4}{(s-4m_\pi^2)^2} \sum_{l=0}^{\infty} (2l+1) P_l''(\cos \theta) \text{Im } f_l(s) < (\text{Const.}) s^2.$$

We can now use this to bound the sum  $\sum_{l=l_0}^{\infty}$  in the expression for  $\text{Im } F$ , and the unitarity bound  $\text{Im } f_l \leq 2s^{1/2}/\pi k$  for the piece  $\sum_{l=0}^{l_0}$ . Optimizing  $l_0$  (which one takes  $l_0 \sim s^{1/2}$ ) produces the bound (1.1b); the details may be found in ref. 4. This bound improves the standard Froissart bound in (1.1a), that can be obtained from (3.1), because  $P_l''(\cos \theta) \gg P_l'(\cos \theta) \gg P_l(\cos \theta)$ , for  $t \geq 0$ , large  $l$ . This bound (1.1b) is optimal, in the sense that, if we take a further derivative, the corresponding integral,

$$\int_{4\mu^2}^{\infty} ds \frac{\partial^3 \text{Im } F_{\pi^0\pi^+}(s, t)/\partial t^3|_{t=4m_\pi^2}}{s^3},$$

necessarily diverges.

This suggests a behaviour like (1.1b), that is,

$$\sigma_{\pi^0\pi^+}(s) \underset{s \rightarrow \infty}{\simeq} (\text{Const.}) \times \log^2 \frac{s}{s_1 \log^{\frac{7}{2}} s / s_2}. \quad (3.3)$$

#### 4. Phenomenology

In order to compare our result with very high energy  $\pi N$ ,  $KN$  and (especially)  $NN$  cross sections, for which we have good data, we may use factorization to write

$$\begin{aligned} \sigma_{\pi\pi}^{(I_t=0)} &\underset{s \text{ large}}{\simeq} \frac{4\pi^2}{\lambda^{1/2}(s, m_\pi^2, m_\pi^2)} [P(s, 0) + P'(s, 0)], \\ \frac{\sigma_{pp} + \sigma_{\bar{p}p}}{2} &\underset{s \text{ large}}{\simeq} \frac{4\pi^2}{\lambda^{1/2}(s, m_p^2, m_p^2)} \frac{1}{2} f_{N/\pi}^2 [P(s, 0) + (1 + \epsilon) P'(s, 0)], \\ \sigma_{\pi^\pm p} &\underset{s \text{ large}}{\simeq} \frac{4\pi^2}{\lambda^{1/2}(s, m_\pi^2, m_p^2)} f_{N/\pi} \left\{ \frac{1}{\sqrt{6}} [P(s, 0) + P'(s, 0)] \mp \frac{1}{2} \bar{\rho}(s, 0) \right\}, \\ \left. \frac{d\sigma(\pi^- p \rightarrow \pi^0 n)}{dt} \right|_{t=0} &\underset{s \text{ large}}{\simeq} f_{N/\pi}^2 \frac{1 - \cos \pi \alpha_\rho}{\sin^2 \pi \alpha_\rho} \frac{\pi^3}{\lambda(s, m_\pi^2, m_p^2)} |\bar{\rho}(s, 0)|^2 \\ \sigma_{K^+p} + \sigma_{K^-p} &\underset{s \text{ large}}{\simeq} \frac{4\pi^2}{\lambda^{1/2}(s, m_K^2, m_p^2)} f_{N/\pi} f_{K/\pi} [P(s, 0) + r P'(s, 0)]. \end{aligned} \quad (4.1a)$$

Moreover,

$$\bar{\rho}(s, 0) = \bar{\beta}_\rho (s/\hat{s})^{\alpha_\rho(0)}, \quad P'(s, 0) = \beta_{P'} (s/\hat{s})^{\alpha_{P'}(0)}. \quad (4.1b)$$

The quantities  $\alpha_\rho(0)$ ,  $\bar{\beta}_\rho$  have been determined with precision to be<sup>[9]</sup>

$$\bar{\beta}_\rho = \alpha_{P'} = 0.39 \pm 0.02, \quad \alpha_\rho(0) = 0.52 \pm 0.03. \quad (4.1b)$$

One also has  $\epsilon = 0.24$  and  $r$  is very small.

*The extended Regge case.* We have shown before that the assumption of an extended Regge behaviour leads to a growth of cross sections less than  $\log s$ . This is not sufficient to reproduce the rise of the cross sections for hadronic processes observed to occur in the multi-TeV region. We may verify this if, in the extended Regge case, we take the most favourable situation in which the bound (2.6a) is saturated and thus write

$$P(s, 0) = \left\{ a \log \frac{s}{\hat{s}} + \tilde{\beta}_P \right\} s, \quad \hat{s} \equiv 1. \quad (4.2)$$

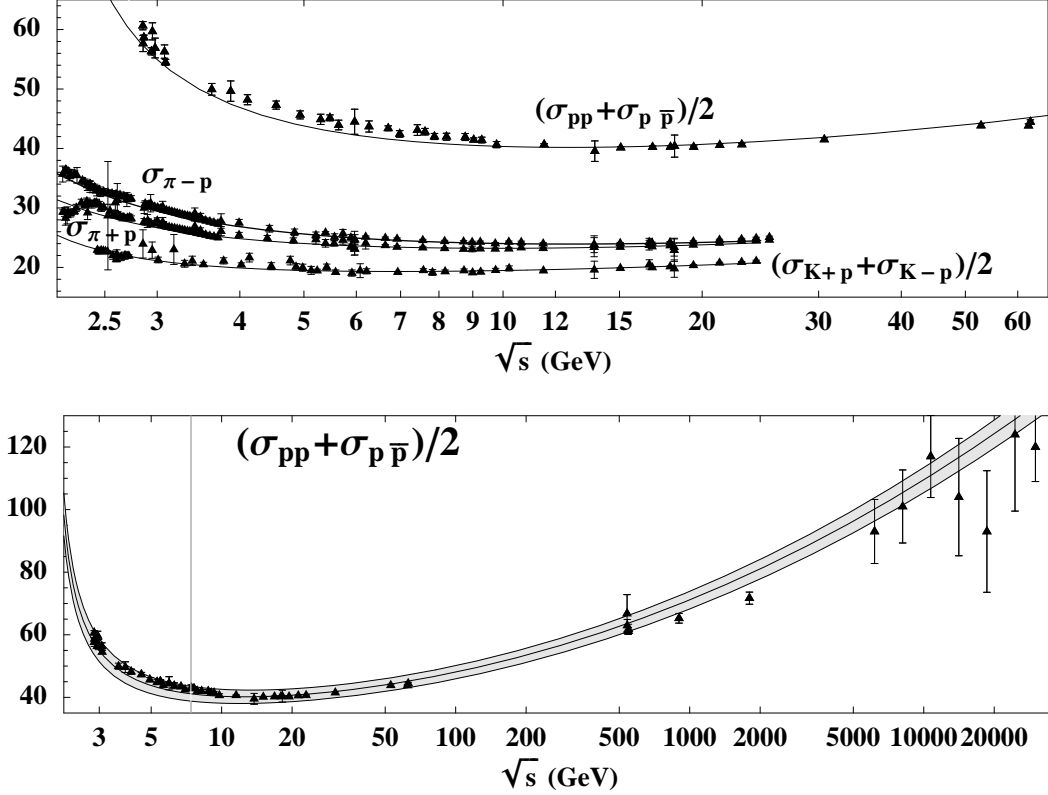


FIGURE 1. The total cross section  $(\sigma_{\bar{p}p} + \sigma_{pp})/2$ . Black dots, triangles and squares: experimental points. For energies above 30 GeV, we have given the experimental values of  $(\sigma_{\bar{p}p} + \sigma_{pp})/2$  as if they equaled  $\sigma_{\bar{p}p}$  or  $\sigma_{pp}$ . Continuous lines: fits with the saturated Froissart bound hypothesis.

With this, we fit data for  $\pi\pi$ ,  $\pi^\pm p$ ,  $K^+p + K^-p$ , and  $pp + \bar{p}p$  cross sections<sup>2</sup> from a kinetic energy in the c.m. of one GeV up to the highest energies attained, 30 TeV in cosmic ray experiments.<sup>[10]</sup> We find

$$a = 0.413 \text{ mb}, \quad \tilde{\beta}_P = -1.36, \quad f_{N/\pi} = 1.320, \quad f_{K/\pi} = 0.831. \quad (4.3)$$

The fit is not very good, as one has  $\chi^2/\text{d.o.f.} = 1.29$ . The value of the corresponding cross section at the LHC is as reported in (1.6), certainly too low.

*The saturated Froissart bound case.* The *saturated bound* hypothesis fares better. We write now,

$$P(s, 0) = \left\{ A \log^2 \frac{s}{s_1 \log^{\frac{7}{2}} s/s_2} + \tilde{\beta}_P \right\} s. \quad (4.4)$$

The details of the fit may be found in ref. 9. We choose the fit called “fit C” there and have

$$\begin{aligned} f_{N/\pi} &= 1.359 \pm 0.004, \quad \tilde{\beta}_P = 2.32 \pm 0.04, \\ A &= 0.033 \pm 0.001, \quad s_1 = 0.01 \text{ GeV}^2, \quad s_2 = 0.15 \pm 0.05 \text{ GeV}^2. \end{aligned} \quad (4.5)$$

The fit has chi-squared of  $\simeq 1.2$  which, for almost 500 experimental points, is clearly better than that with the extended Regge hypothesis (particularly if we realize that a  $\chi^2/\text{d.o.f.}$  of 1.15 would be obtained if relaxing the extended factorization hypothesis, and a  $\chi^2/\text{d.o.f.}=1$  would follow if excluding  $\pi^+p$  data for  $s^{1/2} < 3$  GeV; cf. ref. 9). The fit is depicted, for some of the processes, in Fig. 1.

<sup>2</sup> For details on the choice of data, errors, etc., see ref. 9.

## 5. Discussion

The fact that it is impossible to get a good fit with the parametrization (4.2) in the whole energy range  $1 \text{ GeV} \lesssim E_{\text{kin}} \lesssim 30 \text{ TeV}$ , while (4.4) produces a clearly better fit in the same energy region, suggests that it is the saturated Froissart bound hypothesis, and not the extended Regge one, that makes the ultra high energy behaviour of cross sections compatible with what one finds at (relatively) lower energies. That the extended Regge behaviour fails means that the behaviour (2.4) must only hold for values of  $t$  near zero. In fact, it is not difficult to realize that the behaviour (1.1b) is only compatible with the saturated Froissart bound, Eq. (3.3), under the following conditions: the function  $\alpha_s(t)$  in (1.2) must flatten out before vanishing. Moreover, if we call  $t_0$  to the point where  $\alpha_s(t)$  first vanishes, the residue function  $\Phi(t)$  must change sign at relatively small values of  $t$  and continue negative until  $t \sim t_0$  and, furthermore,  $t_0$  must be of the order of  $s$ , for large  $s$ . This is so because one must cancel, to a relative precision  $O(1/\log s)$ , the integral

$$\int_{-t_1}^0 dt P_l(\cos \theta) \text{Im } F_{\pi^0 \pi^+}(s, t) \sim \log s$$

with the remainder,

$$\int_{4m_\pi^2 - s}^{-t_1} dt P_l(\cos \theta) \text{Im } F_{\pi^0 \pi^+}(s, t),$$

where  $t_1$  is the first zero of  $\Phi(t)$ . Now, since  $\alpha_P(t)$  is assumed to be independent of  $s$ , it follows that one should have  $\alpha_P(t) > 0$  for all  $t$ . In particular, it follows that the transition from the Regge behaviour to the Brodsky–Farrar one would be very rough, involving violent oscillations of the scattering amplitude. Indeed, to get from a behaviour  $\text{Im } F_{\pi^0 \pi^+} \sim \Phi s^0$  to  $\text{Im } F_{\pi^0 \pi^+} \sim s^{-6}$  one should have very strong oscillations of  $\Phi(t)$  which would average to zero when  $|t| \rightarrow \infty$ .

This is very unlikely; what probably happens is that the classical Regge-type expression,<sup>[11]</sup>

$$P(s, t) = a(\log^\nu s / \hat{s}) \alpha_P(t) \frac{1 + \alpha_P(t)}{2} e^{bt(s/\hat{s})^{\alpha_P(t)}}, \quad \alpha_s(t) \simeq 1 + \alpha'_P(0)t,$$

fails well before the point  $t_0$  where  $\alpha_P(t_0) = 0$ , i.e., well below  $|t| \sim 5 \text{ GeV}^2$ .



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